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1996 J. Phys. A: Math. Gen. 29 1605

(http://iopscience.iop.org/0305-4470/29/8/009)

J. Phys. A: Math. Gen. **29** (1996) 1605–1618. Printed in the UK

# Replica-symmetry breaking and quantum fluctuation effects in the $p$-spin interaction spin-glass model with a transverse field

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Received 20 June 1995, in final form 13 November 1995

**Abstract.** The Ising infinite-range spin-glass model with $p$-spin interactions in the presence of a transverse field is studied for large but finite $p$ using the Matsubara time representation and Parisi's scheme of replica-symmetry breaking. In the spin-glass phase, the corrections to the limit $p \to \infty$ appear much more essential than in the classical counterpart. It is shown that the quantum fluctuations have the effect of destroying the spin-glass order and a lower temperature is required to stabilize the spin-glass phase. The spin autocorrelation function in the spin-glass phase is explicitly calculated as a function of $p$ and the Matsubara time. The result is just complementary and consistent with that previously obtained for the paramagnetic state without using the replica method.

## 1. Introduction

The Ising infinite-range spin-glass (SG) model with $p$-spin interactions [1–6] is a good theoretical laboratory for exploring some crucial aspects which are inaccessible for real SGs. As is known, the classical model is exactly solvable in the limit $p \to \infty$ [1] and is equivalent to the random energy model (REM) [2]. In this limit, it is sometimes referred to as 'the simplest SG'. However, studies have also been made [3] for large but finite $p$ and around $p = 2$ (the usual SG Ising model). More recently, the quantum version of this model in the presence of a transverse field has attracted some attention [4–6] with the main purpose being to investigate the effective role played by the quantum fluctuations. This aspect of the SG problem appears difficult to study in a reliable way for real quantum spin-glasses [7] and the existence of a non-trivial but simple quantum SG model is very fortunate so that exact analytical information can be accessible. Some studies have been achieved using the replica method [8] via the Suzuki–Trotter transformation [9] for casting the transverse field quantum model into an equivalent classical one. An interesting picture has been pointed out for $p \to \infty$ [4], within the static approximation (SA) assumed exact in this limit and for large but finite $p$ [5]. In the last case, corrections to the $p \to \infty$ SA results have been obtained and the effects of the finite $p$ changes on the phase diagram have also been explored. Briefly, the main results are as follows.

(i) In the limit $p \to \infty$, the phase diagram consists of three phases [4], a SG phase and two paramagnetic phases distinguished by a transverse ordering. In particular, in the

0305-4470/96/081605+14$19.50  

paramagnetic state, it is found that the system exhibits a classical paramagnetic (CP) phase, where the quantum fluctuations are irrelevant, and a quantum paramagnetic (QP) phase where the quantum spins appear non-interacting.

(ii) For large but finite $p$, the paramagnetic state scenario is found to change drastically [5]. A new critical point occurs, ending the transition line between the two paramagnetic phases, and the quantum fluctuations appear to have a more relevant role.

Similar results for the paramagnetic state have been recently achieved [6] by means of a quantum version of the so-called 'cavity fields approach' [10]. In particular, the large-$p$ corrections for the dynamical spin autocorrelation function have been obtained analytically in explicit form. Here, replicas and the Suzuki–Trotter transformation are avoided and the non-commutativity of the spin operators is conveniently taken into account using the Matsubara time representation [11].

Less attention has been devoted to the SG phase when the quantum degrees of freedom are switched on. For this phase, some information exists only in the limit $p \to \infty$ [4] where the quantum fluctuations are found to be completely ineffective. Therefore, the understanding of the effective role played by quantum fluctuations in the SG phase for $p < \infty$ remains an open problem and one can reasonably hope that a reliable study of the most accessible large-$p$ case might also give additional insight about the relevance of the quantum effects in realistic SG models (remember that $p = 2$ corresponds to the Ising model in a transverse field) for which a large amount of information has been recently acquired [7]. However, it is worth mentioning that consistent predictions for the large-$p$-spin interaction SG model have, in any case, generated intrinsic interest in the light of a recent proposal [12, 13] to use this model for solving the problem of optimal coding in the transmission of information.

The purpose of the present paper is to explore analytically the SG phase of the mentioned quantum model for large but finite $p$, with particular attention to quantum fluctuation effects. Here we use the replica method within the Matsubara time representation [6, 11] for a direct extension of Parisi's scheme of replica-symmetry breaking [10].

The paper is organized as follows. In section 2 we introduce the Matsubara time representation for the model and use the replica method as a direct extension of the known classical picture for describing the SG phase. Section 3 is devoted to Parisi's scheme of replica-symmetry breaking. Here, the self-consistent equations for the autocorrelation function and for Parisi's parameters are obtained. The relevant analytical results for large $p$ are presented in section 4. Finally, some concluding remarks are drawn in section 5. An appendix is also added and contains some details about the non-standard calculation of integrals which are involved in the main text.

## 2. Replica method and saddle-point self-consistent equations

We consider the quantum SG model with the Hamiltonian [4–6]

$$H = -\sum_{i_1 \ldots i_p}^{N} J_{i_1 \ldots i_p} \sigma_{i_1}^z \cdots \sigma_{i_p}^z - \Gamma \sum_{i=1}^{N} \sigma_i^x \tag{1}$$

where the sum $(i_1 \ldots i_p)$ runs over all distinct clusters of $p$ spins, $\sigma_i^z$ and $\sigma_i^x$ are the Pauli matrices at site $i$, $N$ is the total number of sites and $\Gamma > 0$ denotes the strength of the transverse field. The random couplings $J_{i_1 \ldots i_p}$ are distributed according to

$$P(J_{i_1 \ldots i_p}) = (N^{p-1}/J^2 \pi p!) \exp\left[ -\frac{(J_{i_1 \ldots i_p})^2 N^{p-1}}{J^2 p!} \right]. \tag{2}$$

For $\Gamma = 0$ the Hamiltonian (1) describes, in the limit $p \to \infty$, the REM [1, 2].

In order to perform averages over quenched couplings $J$ and to calculate the free energy of the model, we use the replica method with the introduction of the Matsubara time representation [11] avoiding the Suzuki–Trotter transformation [9]. The free energy of the system reads

$$-\beta F = [\ln \mathrm{Tr}\, \mathrm{e}^{\beta H}]_{\mathrm{av}} = \lim_{n\to 0} \ln \mathrm{Tr}\left[\exp\left(-\beta \sum_{\alpha=1}^{n} H^{(\alpha)}\right)\right]_{\mathrm{av}} \tag{3}$$

where $\beta = 1/k_{\mathrm{B}}T$, $k_{\mathrm{B}}$ is the Boltzmann constant (here assumed equal to unity) and $T$ is the temperature. In equation (3), $H^{(\alpha)}$ is the $\alpha$th replica of the Hamiltonian (1), $n$ denotes the replica number and

$$[\cdots]_{\mathrm{av}} = \int_{-\infty}^{\infty} \prod_{i_1\ldots i_p}^{N} P(J_{i_1\ldots i_p})\, \mathrm{d}J_{i_1\ldots i_p}. \tag{4}$$

At this stage, we introduce the Matsubara time representation [6, 11] in which the $\tau$-ordering operation $T_\tau$ makes the handling of the operators as $c$-numbers possible. Within the interaction representation, we can write

$$\exp\left(-\beta \sum_{\alpha=1}^{n} H^{(\alpha)}\right) = \mathrm{e}^{-\beta H_0} T_\tau \exp\left[-\int_0^\beta \mathrm{d}\tau\, H_1(\tau)\right] \tag{5}$$

with

$$H_0 = \sum_{\alpha=1}^{n} H_0^{(\alpha)} \tag{6}$$

$$H_1 = \sum_{\alpha=1}^{n} H_1^{(\alpha)} \tag{7}$$

and

$$H_1(\tau) = \mathrm{e}^{\tau H_0} H_1 \mathrm{e}^{-\tau H_0} \tag{8}$$

where $H_0^{(\alpha)}$ and $H_1^{(\alpha)}$ denote the $\alpha$th replicas of $-\Gamma \sum_{i=1}^{N} \sigma_i^x$ and $-\sum_{i_1\ldots i_p}^{N} J_{i_1\ldots i_p} \sigma_{i_1}^z \cdots \sigma_{i_p}^z$, respectively. Then, (3) can be rewritten as

$$-\beta F = \lim_{n\to 0} \ln Z_n \tag{9}$$

with

$$Z_n = \mathrm{Tr}\left\{\mathrm{e}^{-\beta H_0} T_\tau \exp\left[\frac{NJ^2}{4} \sum_{\alpha,\alpha'=1}^{n} \int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\, \hat{Q}^p_{\alpha,\alpha'}(\tau,\tau';[\sigma]) + \mathcal{O}(1/N)\right]\right\}. \tag{10}$$

In this equation,

$$\hat{Q}_{\alpha,\alpha'}(\tau,\tau';[\sigma]) = \frac{1}{N} \sum_{i=1}^{N} \sigma_{i\alpha}^z(\tau) \sigma_{i\alpha'}^z(\tau') \tag{11}$$

with

$$\sigma_{i\alpha}^z(\tau) = \mathrm{e}^{-\tau\Gamma\sigma_{i\alpha}^x} \sigma_{i\alpha}^z \mathrm{e}^{\tau\Gamma\sigma_{i\alpha}^x} \tag{12}$$

where $\sigma_{i\alpha}^x$, $\sigma_{i\alpha}^z$ denote the $\alpha$th replicas of $\sigma_i^x$ and $\sigma_i^z$, respectively.

The spin trace in equation (10) can be performed by constraining $\hat{Q}_{\alpha,\alpha'}(\tau,\tau';[\sigma])$ to be equal to a $c$-number function $Q_{\alpha\alpha'}(\tau,\tau')$ using an appropriate Lagrange-multiplier matrix $\mu_{\alpha,\alpha'}(\tau,\tau')$. Then one obtains the functional representation

$$Z_n = \int \mathcal{D}[Q]\mathcal{D}[\mu]\mathrm{e}^{-N\mathcal{H}[Q,\mu]} \tag{13}$$

with

$$\begin{aligned}\mathcal{H}[Q,\mu] = &-\frac{J^2}{4}\sum_{\alpha,\alpha'=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\, Q^p_{\alpha,\alpha'}(\tau-\tau') \\ &+\frac{J^2}{2}\sum_{\alpha,\alpha'=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\, Q_{\alpha,\alpha'}(\tau-\tau')\mu_{\alpha\alpha'}(\tau-\tau') - \ln\mathrm{Tr}\left\{\mathrm{e}^{-\beta\sum_{\alpha=1}^{n}H_0^{(\alpha)}}T_\tau\right. \\ &\left.\times\exp\left[\frac{J^2}{2}\sum_{\alpha,\alpha'=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\, \mu_{\alpha\alpha'}(\tau-\tau')\sigma^z_\alpha(\tau)\sigma^z_{\alpha'}(\tau')\right]\right\}\end{aligned} \tag{14}$$

where $\sigma^z_\alpha$ denotes the operator $\sigma^z_{i\alpha}$ for an arbitrary site. In writing equation (14) we have assumed the translational symmetry in the Matsubara time direction ($Q_{\alpha,\alpha'}$ and $\mu_{\alpha,\alpha'}$ depend on the difference $(\tau-\tau')$).

In the thermodynamic limit $\mathrm{N}\to\infty$, taking the saddle point of $Q$- and $\mu$-functional integrals in equation (13), the free energy (9) is given by

$$\frac{\beta F}{N} = \lim_{n\to 0}\frac{\mathcal{H}}{n} \tag{15}$$

where $\mathcal{H}\equiv\mathcal{H}[Q,\mu]$, valued in the saddle-point solution for $Q$ and $\mu$. For future practical purposes, we find it convenient to separate the parameters $Q_{\alpha,\alpha'}$, and $\mu_{\alpha,\alpha'}$ into diagonal and non-diagonal setting

$$\chi(\tau-\tau') = Q_{\alpha\alpha}(\tau-\tau') \tag{16}$$

$$q_{\alpha\alpha'}(\tau-\tau') = Q_{\alpha\alpha'}(\tau-\tau') \qquad \text{for } \alpha\neq\alpha' \tag{17}$$

$$\nu(\tau-\tau') = \mu_{\alpha\alpha}(\tau-\tau') \tag{18}$$

and

$$\lambda_{\alpha\alpha'}(\tau-\tau') = \mu_{\alpha\alpha'}(\tau-\tau') \qquad \text{for } \alpha\neq\alpha'. \tag{19}$$

Then, without any static ansatz, for $\mathcal{H}/n$ in (15) we can write

$$\begin{aligned}\frac{\mathcal{H}}{n} = &-\frac{J^2}{4}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\chi^p(\tau-\tau') - \frac{J^2}{4n}\sum_{\alpha\neq\alpha'=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\, q^p_{\alpha\alpha'}(\tau-\tau') \\ &+\frac{J^2}{2}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\chi(\tau-\tau')\nu(\tau-\tau') \\ &+\frac{J^2}{2n}\sum_{\alpha\neq\alpha'=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\, q_{\alpha\alpha'}(\tau-\tau')\lambda_{\alpha\alpha'}(\tau-\tau') \\ &-\frac{1}{n}\ln\mathrm{Tr}\left\{\mathrm{e}^{-\beta H_0}T_\tau\exp\left[\frac{J^2}{2}\sum_{\alpha=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\nu(\tau-\tau')\sigma^z_\alpha(\tau)\sigma^z_\alpha(\tau')\right.\right. \\ &\left.\left.+\frac{J^2}{2}\sum_{\alpha\neq\alpha'=1}^{n}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\lambda_{\alpha\alpha'}(\tau-\tau')\sigma^z_\alpha(\tau)\sigma^z_{\alpha'}(\tau')\right]\right\}.\end{aligned} \tag{20}$$

Of course, the parameters $\chi(\tau-\tau')$, $q_{\alpha\alpha'}(\tau-\tau')$, $\nu(\tau-\tau')$ and $\lambda_{\alpha\alpha'}(\tau-\tau')$ are determined by the extremum conditions

$$\frac{\delta\mathcal{H}}{\delta\chi(\tau-\tau')}=\frac{\delta\mathcal{H}}{\delta q_{\alpha\alpha'}(\tau-\tau')}=0 \tag{21}$$

and

$$\frac{\delta\mathcal{H}}{\delta\nu(\tau-\tau')}=\frac{\delta\mathcal{H}}{\delta\lambda_{\alpha\alpha'}(\tau-\tau')}=0. \tag{22}$$

From equation (11), it is clear that the solutions $\chi(\tau-\tau')$ and $q_{\alpha\alpha'}(\tau-\tau')$ represent the spin autocorrelation function [6] and the SG order parameter function [1, 13], respectively. The quantities $\nu(\tau-\tau')$ and $\lambda_{\alpha\alpha'}(\tau-\tau')$, related to the original Lagrange multipliers $\mu_{\alpha\alpha'}$, are simply auxiliary parameters controlling the constraints for $\hat{Q}_{\alpha,\alpha'}(\tau,\tau';[\sigma])$ in equation (10) and have no direct physical meaning.

From the extremum conditions (21) and (22), it is easy to obtain the following system of coupled self-consistent equations

$$\nu(\tau-\tau')=\frac{p}{2}\chi^{p-1}(\tau-\tau') \tag{23}$$

$$\lambda_{\alpha\alpha'}(\tau-\tau')=\frac{p}{2}q_{\alpha\alpha'}^{p-1}(\tau-\tau') \tag{24}$$

$$\begin{aligned}\chi(\tau-\tau')=&\bigg\langle T_\tau \exp\bigg\{\frac{J^2}{2}\sum_{\alpha_1,\alpha_2=1}^{n}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,[\nu(\tau-\tau')\delta_{\alpha_1\alpha_2}\\&+\lambda_{\alpha_1\alpha_2}(\tau-\tau')]\sigma_{\alpha_1}^z(\tau)\sigma_{\alpha_2}^z(\tau')\bigg\}\sigma_\alpha^z(\tau)\sigma_\alpha^z(\tau')\bigg\rangle_0\\&\times\bigg\langle T_\tau \exp\bigg\{\frac{J^2}{2}\sum_{\alpha_1,\alpha_2=1}^{n}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,[\nu(\tau-\tau')\delta_{\alpha_1\alpha_2}\lambda_{\alpha_1\alpha_2}(\tau-\tau')]\\&\times\sigma_{\alpha_1}^z(\tau)\sigma_{\alpha_2}^z(\tau')\bigg\}\bigg\rangle_0\end{aligned} \tag{25}$$

and

$$\begin{aligned}q_{\alpha\alpha'}(\tau-\tau')=&\bigg\langle T_\tau \exp\bigg\{\frac{J^2}{2}\sum_{\alpha_1,\alpha_2=1}^{n}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,[\nu(\tau-\tau')\delta_{\alpha_1\alpha_2}\\&+\lambda_{\alpha_1\alpha_2}(\tau-\tau')]\sigma_{\alpha_1}^z(\tau)\sigma_{\alpha_2}^z(\tau')\bigg\}\sigma_\alpha^z(\tau)\sigma_{\alpha'}^z(\tau')\bigg\rangle_0\\&\times\bigg\langle T_\tau \exp\bigg\{\frac{J^2}{2}\sum_{\alpha_1,\alpha_2=1}^{n}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,[\nu(\tau-\tau')\delta_{\alpha_1\alpha_2}+\lambda_{\alpha_1\alpha_2}(\tau-\tau')]\\&\times\sigma_{\alpha_1}^z(\tau)\sigma_{\alpha_2}^z(\tau')\bigg\}\bigg\rangle_0^{-1}\end{aligned} \tag{26}$$

where

$$\langle\cdots\rangle_0=\frac{\mathrm{Tr}\,\mathrm{e}^{-\beta H_0}\cdots}{\mathrm{Tr}\,\mathrm{e}^{-\beta H_0}} \tag{27}$$

and the convention $\lambda_{\alpha,\alpha}=0$ is understood. From (25) and (26) it is easy to see that $q_{\alpha\alpha'}(\tau-\tau')\leqslant 1$ and $\chi(\tau-\tau')\leqslant 1$.

## 3. Parisi's scheme of replica-symmetry breaking

Now we are in the position to show that Parisi's anzatz for replica-symmetry breaking (RSB) [10] is valid in our quantum model and a possible self-consistent solution for the SG order parameter function for large $p$ (not necessary in the limit $p \to \infty$) is simply a step function. This means that, similarly as in the classical counterpart [1], the first-order breaking of replica symmetry is exact.

Following the conventional Parisi procedure [10], to the general $k$th order RSB, we introduce a sequence of variables

$$0 \leqslant q_0(\tau-\tau') \leqslant q_1(\tau-\tau') \leqslant \cdots \leqslant q_{k-1}(\tau-\tau') \leqslant q_k(\tau-\tau') \leqslant 1$$

and

$$0 \leqslant \lambda_0(\tau-\tau') \leqslant \lambda_1(\tau-\tau') \leqslant \cdots \leqslant \lambda_{k-1}(\tau-\tau') \leqslant \lambda_k(\tau-\tau') \leqslant 1$$

with $k$ an arbitrary integer, which are related to the parameters $q_{\alpha\alpha'}(\tau-\tau')$ and $\lambda_{\alpha\alpha'}(\tau-\tau')$, respectively. Then, we can rewrite equation (16) in the form [1, 10]

$$\begin{aligned}\frac{\beta F}{N} = \lim_{n\to 0}\frac{\mathcal{H}}{n} = &-\frac{J^2}{4}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,\chi^p(\tau-\tau')\\ &+\frac{J^2}{4}\sum_{l=0}^{k}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,(m_{l+1}-m_l)q_l^p(\tau-\tau')\\ &+\frac{J^2}{2}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,\chi(\tau-\tau')\nu(\tau-\tau')\\ &-\frac{J^2}{4}\sum_{l=0}^{k}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,(m_{l+1}-m_l)q_l(\tau-\tau')\lambda_l(\tau-\tau') - \lim\frac{1}{n}\ln S \end{aligned}\tag{28}$$

where $m_0 = n, m_1, \ldots, m_k, m_{k+1} = 1$ are the tree branch parameters [1, 10] satisfying the inequalities

$$m_0 \leqslant m_1 \leqslant \cdots \leqslant m_{k-1} \leqslant m_k \leqslant 1$$

as $n \to 0$. In expression (28),

$$S = G_0\left(G_1\left(\cdots\left(G_{k-1}(G_k Z_0^{m_k}[h])^{m_{k-1}/m_k}\right)^{m_{k-2}/m_{k-1}}\cdots\right)^{m_1/m_2}\right)^{n/m_1}\Bigg|_{h(\tau)=0}\tag{29}$$

where the functional differential operators $G_l$ $(l = 0, 1, \ldots, k)$ are defined by

$$G_l = \exp\left\{\frac{J^2}{2}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,[\lambda_l(\tau-\tau') - \lambda_{l-1}(\tau-\tau')]\frac{\delta^2}{\delta h(\tau)\delta h(\tau')}\right\}\tag{30}$$

with $\lambda_{-1} = 0$ when $l = 0$, and

$$Z_0[h] = \mathrm{Tr}\left(\mathrm{e}^{\beta\Gamma\sigma^x}T_\tau\left\{U(\beta,0)\exp\left[\int_0^\beta \mathrm{d}\tau\,h(\tau)\sigma^z(\tau)\right]\right\}\right)\tag{31}$$

with

$$U(\beta,0) = T_\tau\left(\exp\left\{\frac{J^2}{2}\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,[\nu(\tau-\tau') - \lambda_k(\tau-\tau')]\sigma^z(\tau)\sigma^z(\tau')\right\}\right).\tag{32}$$

For large $p$, from relation (24), this results in

$$\lambda_l(\tau-\tau') = \frac{p}{2}q_l^{p-1}(\tau-\tau') \approx \frac{p}{2}q_l^p(\tau-\tau')$$

and, if we assume $q_l(\tau-\tau') < 1$ for $l = 0, 1, \ldots, k-1$ and $q_k(\tau-\tau') = 1$ in the limit $p \to \infty$, we have that $\lambda_l(\tau-\tau')$ $(l = 0, 1, \ldots, k-1)$ are very small and $\lambda_k(\tau-\tau')$ is large for large $p$. If, on the contrary, $q_k(\tau-\tau') < 1$ in the limit $p \to \infty$, we would reproduce the replica-symmetric solution [1, 3]. Then, expanding the quantity $S$, defined by equation (29), to the first order in $\lambda_{l<k}(\tau-\tau')$, for large $p$ we get the free energy in the form

$$\begin{aligned}\frac{\beta F}{N} = \lim_{n\to 0}\frac{\mathcal{H}}{n} &= -\frac{J^2}{4}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\chi^p(\tau-\tau') + \frac{J^2}{2}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\chi(\tau-\tau')\nu(\tau-\tau') \\ &\quad + \frac{J^2}{4}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,(1-m_k)q_k^p(\tau-\tau') \\ &\quad - \frac{J^2}{2}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,(1-m_k)q_k(\tau-\tau')\lambda_k(\tau-\tau') \\ &\quad - \frac{1}{m_k}\ln I_k - \ln 2 + \mathcal{O}(\lambda_{l<k}\lambda_{l'<k}).\end{aligned} \tag{33}$$

In equation (33),

$$I_k = I_k[h]|_{h(\tau)=0} \tag{34}$$

with

$$I_k[h] = 2^{-m_k}\tilde{G}_k Z_0^{m_k}[h] \tag{35}$$

and

$$\tilde{G}_k = \exp\left[\frac{J^2}{2}\int_0^\beta \mathrm{d}\tau \int_0^\beta \mathrm{d}\tau'\,\lambda_k(\tau-\tau')\frac{\delta^2}{\delta h(\tau)\delta h(\tau')}\right].$$

As we see from equation (33), the linear terms in $\lambda_{l<k}(\tau-\tau')$ are not present (they would appear only if $h(\tau) \neq 0$). Therefore the extremum condition

$$\frac{\delta F}{\delta\lambda_{l<k}(\tau-\tau')} = 0 \tag{36}$$

is satisfied for $q_{l<k}(\tau-\tau') = 0$. At this stage, it is interesting to note that the solution $q_{l<k}(\tau-\tau') = 0$ of equation (36), under the conditions assumed above for $\lambda_{l<k}(\tau-\tau')$, is a trivial solution of the extremum condition with $F$ not only approximated by (33), but also in the exact form (28). Such a type of solution exists formally for all values of $p$, but it leads to a physically acceptable result only in the limit $p \to \infty$ and for large $p$, a situation which occurs also for the classical counterpart of our model with $\Gamma = 0$ [1, 3]. For $p = 2$ and $\Gamma = 0$, it can be shown that the solutions $q_{l<k} = 0$ are related to unphysical values of $q_k > 1$ and $m_k > 1$. In this case, only the solutions $q_{l<k} > 0$ are physically acceptable giving for $k \to \infty$ the Parisi order parameter function $q(x)$ [10]. For our quantum model, the calculation of the non-trivial solutions with $q_{l<k} > 0$ for arbitrary $p$ is a rather difficult task (even for large $p$) since it requires the solution of equation (36) with the exact expression (28) for $F$.

Limiting ourselves to consider the free energy expansion (33), it is easy to obtain general expressions for the spin autocorrelation function $\chi(\tau-\tau')$ and the SG order parameter $q_k(\tau-\tau')$, which are valid for large $p$. In section 4, we will give explicit results.

The extremum conditions

$$\frac{\delta F}{\delta\nu(\tau-\tau')} = 0 \qquad \frac{\delta F}{\delta\lambda_k(\tau-\tau')} = 0 \tag{37}$$

yield

$$\chi(\tau-\tau') = \frac{2^{-m_k}\tilde{G}_k Z_0^{m_k-1}[h](\delta^2 Z_0[h]/\delta h(\tau)\delta h(\tau'))|_{h(\tau)=0}}{I_k} \tag{38}$$

and

$$q_k(\tau-\tau') = \frac{2^{-m_k}\tilde{G}_k Z_0^{m_k-2}[h](\delta Z_0[h]/\delta h(\tau))(\delta Z_0[h]/\delta h(\tau'))|_{h(\tau)=0}}{I_k}. \tag{39}$$

Now, it is known [4, 5] that the SA is valid for the SG phase in the limit $p \to \infty$ and under such a condition one has [4] $\chi(\tau-\tau') = q_k(\tau-\tau') = 1$, whereas $\nu(\tau-\tau') = \lambda_k(\tau-\tau') = p/2$ for large $p$. Then, in order to calculate the large-$p$ expressions for $\chi(\tau-\tau')$ and $q_k(\tau-\tau')$ from equations (38) and (39), respectively, it is sufficient to put $\lambda_k(\tau-\tau') = \nu(\tau-\tau') = p/2$ in their right-hand sides. Since $U(\beta, 0) = 1$, $Z_0[h]$ (see equation (31)) simplifies considerably and, with straightforward calculations, we get

$$\begin{aligned}\chi(\tau-\tau') = 1 - \frac{\Gamma^2}{I_k}\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}\mathrm{e}^{-x^2/2}\frac{\cosh^{m_k}[\beta E_0(x)]}{E_0^2(x)} \\ \times\left\{\tanh[\beta E_0(x)] - \frac{\cosh[(\beta-2|\tau-\tau'|)E_0(x)]}{\cosh[\beta E_0(x)]}\right\}\end{aligned} \tag{40}$$

and

$$q_k(\tau-\tau') = \frac{pJ^2}{2I_k}\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}x^2\mathrm{e}^{-x^2/2}\frac{\cosh^{m_k}[\beta E_0(x)]}{E_0^2(x)}\tanh^2[\beta E_0(x)] \tag{41}$$

where

$$I_k = \int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}\mathrm{e}^{-x^2/2}\cosh^{m_k}[\beta E_0(x)] \tag{42}$$

and

$$E_0(x) = \sqrt{\Gamma^2 + \frac{pJ^2}{2}x^2}. \tag{43}$$

## 4. Explicit results for relevant quantities in the spin-glass phase

For obtaining explicit large-$p$ results for $\chi(\tau-\tau')$ and $q_k(\tau-\tau')$, it is necessary to calculate the integrals which appear in equations (40)–(42).

Considering the integral in equation (41) we can use the saddle-point method taking for $m_k$ the known exact result $m_k = 2\sqrt{\ln 2}/(\beta J)$ for $p \to \infty$ [1, 3–5]. Then, from equation (41), one obtains for $q_k(\tau-\tau')$ the large-$p$ expression

$$q_k(\tau-\tau') = q_k = 1 - \frac{4\Gamma^2 T_\mathrm{c}^2}{J^4 p^2} \tag{44}$$

where

$$T_\mathrm{c} = \frac{J}{2\sqrt{\ln 2}} \tag{45}$$

is the SG transition temperature in the limit $p \to \infty$. Result (44) shows in an explicit way that, starting from the SA, one obtains no dependence of the Matsubara time for the SG parameter $q_k$. From the physical point of view, this seems to be justified since the sequence of the parameters $q_0, q_1, \ldots, q_k$ is related to the overlaps between pure states defined as [10]

$$q_{ab} = \frac{1}{N} \sum_{i=1}^{N} m_i^a m_i^b \tag{46}$$

where $m_i^a$ and $m_i^b$ are the local magnetizations along the $z$-direction in the pure states $a$ and $b$, respectively. Obviously, in the case of the quantum system, definition (46) shows that $q_{ab}$ does not depend on the Matsubara time. In principle, equation (39) may give rise to a $(\tau - \tau')$-dependence, but a clear response to this question requires the solution of (39) in an unperturbative manner, i.e. without using the static theory as the zeroth-order approximation. However, such a procedure is quite difficult and analytically unfeasible. Only numerical calculations are possible but this is beyond our present purpose.

For calculation of the spin autocorrelation function $\chi(\tau - \tau')$ from equation (40), it is necessary to find, for large $p$, the breakpoint $m_k$ for Parisi's SG order parameter function. This can be realized starting from the extremum condition

$$\frac{\partial F}{\partial m_k} = 0 \tag{47}$$

where $F$ is given by equation (33). Using the SA expression (42) for $I_k$ valued by the saddle-point method (see equation (A10) of the appendix), the stationary condition (47) for large $p$ becomes

$$\frac{\ln 2}{m_k^2} - \frac{(\beta J)^2}{4} q_k^p = 0. \tag{48}$$

Hence, we get immediately

$$m_k = \frac{2\sqrt{\ln 2}}{\beta J} q_k^{-p/2} = \frac{T}{T_c} \left(1 - \frac{4\Gamma^2 T_c^2}{J^4 p^2}\right)^{-p/2} \approx \frac{T}{T_c(p)} \tag{49}$$

where

$$T_c(p) = T_c \left(1 - \frac{2\Gamma^2 T_c^2}{J^4 p}\right) \tag{50}$$

with $T_c$ given by (45). From equations (49) and (50) we see that, for large but finite $p$, the SG transition temperature $T_c(p)$ is smaller than $T_c$ for $p \to \infty$. This means that the quantum fluctuations, which are irrelevant in the model for $p \to \infty$, have the effect of destroying the SG order for finite $p$ so that the quantum SG phase stabilizes at a lower temperature with respect to the classical counterpart.

Notice that our equations (44) for $q_k$ and (50) for the transition temperature do not reproduce the known results for the classical model for large $p$ when $\Gamma = 0$ [3], but only for $p \to \infty$. This is a strict consequence of using the saddle-point method to calculate the integral over $x$ in (41), which only gives the leading correction for large $p$ with the loss of any higher-order effect. Unfortunately, a full systematic expansion for large $p$ in the presence of a transverse field is very difficult and practically unfeasible. However, one can see immediately that the calculation of $q_k$ and $T_c(p)$ from (41) and (47), respectively, with $\Gamma = 0$ reproduces exactly the results obtained in [3]. So, equation (50) must be modified in

$$T_c(p) = T_c^{\text{class}}(p) \left(1 - \frac{2\Gamma^2 T_c^2}{J^4 p}\right) \tag{51}$$

where, for large $p$ [3],

$$T_c^{\text{class}}(p) = \frac{J}{2\sqrt{\ln 2}}\left[1 + 2^{-p}\sqrt{\frac{\pi}{p(\ln 2)^3}}\right]. \tag{52}$$

So, for the classical model ($\Gamma = 0$), $T_c^{\text{class}}(p)$ increases as $p$ decreases in contrast to what happens for the quantum model ($\Gamma \neq 0$).

Now, we are in the position to obtain the spin autocorrelation function $\chi(\tau - \tau')$ from equation (40). The calculation for large $p$ is now more complicated than that for equation (41), since the integral over $x$ in the last term on the right-hand side of equation (40) cannot be calculated by the saddle-point method for $|\beta - 2|\tau - \tau'|| + \beta_c - \beta \leqslant 0$, with $\beta_c = 1/T_c$. Indeed, the $(\tau - \tau')$-dependent part of $\chi(\tau - \tau')$ in equation (40), with $m_k = \beta_c/\beta$, takes the following form for large $p$:

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}\,\mathrm{e}^{-x^2/2}\cosh^{m_k}[\beta E_0(x)]\frac{\cosh[(\beta - 2|\tau - \tau'|)E_0(x)]}{E_0^2(x)\cosh[\beta E_0(x)]} \approx \int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}\,\mathrm{e}^{f(x)} \tag{53}$$

where

$$f(x) = (|\beta - 2|\tau - \tau'|| + \beta_c - \beta)J\lambda_k|x| - \ln E_0^2(x) - \frac{x^2}{2}. \tag{54}$$

So, in contrast to the case $|\beta - 2|\tau - \tau'|| + \beta_c - \beta > 0$, for $|\beta - 2|\tau - \tau'|| + \beta_c - \beta \leqslant 0$ the function $f(x)$ has no extrema and therefore, for these values of $|\tau - \tau'|$, one cannot use the saddle-point method for calculating the integral (53). Nevertheless, an alternative efficient procedure to calculate this integral for large $p$ and $|\beta - 2|\tau - \tau'|| + \beta_c - \beta \leqslant 0$ is presented in the appendix. The final result for $\chi(\tau - \tau')$ is

$$\chi(\tau - \tau') = \chi_0(p) + \begin{cases} \left\{4\Gamma T_c^2\exp\left[-\frac{pJ^2}{4}(|\beta - 2|\tau - \tau'|| - \beta)\right.\right. \\ \quad\left.\left.\times(\beta - 2\beta_c - |\beta - 2|\tau - \tau'||)\right]\right\} \\ \quad\times[p^2J^4(|\beta - 2|\tau - \tau'|| + \beta_c - \beta)^2]^{-1} \\ \quad\text{for } |\tau - \tau'| < \frac{\beta_c}{2} \text{ and } \beta - \frac{\beta_c}{2} < |\tau - \tau'| < \beta \\ \left\{\Gamma^2\left[(\beta - \beta_c - |\beta - 2|\tau - \tau'||)\ln\left(\frac{\beta - \beta_c - |\beta - 2|\tau - \tau'||}{\beta - \beta_c}\right)\right.\right. \\ \quad\left.\left.+|\beta - 2|\tau - \tau'|| + f(\beta, \Gamma)\right]\exp\left(-\frac{p\beta_c^2J^2}{4}\right)\right\}(\sqrt{\pi p}J)^{-1} \\ \quad\text{for } \frac{\beta_c}{2} \leqslant |\tau - \tau'| \leqslant \beta - \frac{\beta_c}{2} \end{cases} \tag{55}$$

where

$$\chi_0(p) = 1 - \frac{4\Gamma^2T_c^2}{p^2J^4} \tag{56}$$

and the function $f(\beta, \Gamma)$ is defined by equation (A9) of the appendix. It is worth noting that, from definition (25), we must have $\chi(0) = \chi(\pm\beta) = 1$ for any $p$ and equation (55) is just consistent with this condition. From this equation, it is also evident that our calculation for $\chi(\tau - \tau')$ breaks down only for very small values of $p^2J^4(|\beta - 2|\tau - \tau'|| + \beta_c - \beta)^2$.

Now, we have all the ingredients for also calculating the large-$p$ free energy for the SG phase. First notice that, since $q_k(\tau-\tau')$ and $\lambda_k(\tau-\tau')$ do not depend on the $(\tau-\tau')$-parameter, in equation (33) only the integral

$$A=\int_0^\beta \mathrm{d}\tau\int_0^\beta \mathrm{d}\tau'\,\chi^p(\tau-\tau')=\beta\int_0^\beta \mathrm{d}\tau\,\chi^p(\tau). \tag{57}$$

is relevant for large $p$. On the other hand, according to equation (40), we can write for large $p$

$$\chi(\tau)=1-\delta\chi(\tau) \tag{58}$$

with $|\delta\chi(\tau)|\ll 1$. Therefore, for the quantity (57) we have

$$A=\beta\int_0^\beta \mathrm{d}\tau\,[1-\delta\chi(\tau)]^p\approx\beta\int_0^\beta \mathrm{d}\tau\,[1-p\delta\chi(\tau)+\cdots] \tag{59}$$

where $|p\delta\chi(\tau)|\ll 1$ for large $p$ and, in particular, $\lim_{p\to\infty}p\delta\chi(\tau)=0$. So, taking into account that $\delta\chi(\tau)$ is equal to the second term on the right-hand side of (40), with $(\tau-\tau')\to\tau$, we get

$$\int_0^\beta \mathrm{d}\tau\,\delta\chi(\tau)=\frac{\Gamma^2}{I_k}\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}\,\mathrm{e}^{-x^2/2}\frac{\cosh^{m_k}[\beta E_0(x)]}{E_0^2(x)}\tanh[\beta E_0(x)]\left[\beta-\frac{1}{E_0(x)}\right]. \tag{60}$$

Then, inserting expressions (42), (44) and (49) for $I_k$, $q_k(\tau-\tau')$ and $m_k$, respectively, into equation (33), with the help of (57)–(60) and after evaluating the relevant integrals for large $p$ by the saddle-point method, one obtains for the free energy in the SG phase the result

$$\frac{F}{N}=-J\left[\sqrt{\ln 2}-\frac{\Gamma^2}{2pJ^2\sqrt{\ln 2}}+\mathcal{O}\left(\frac{1}{p^2}\right)\right]. \tag{61}$$

As we can see, the free energy in the SG phase for large $p$, similarly as in the limit $p\to\infty$ [4], is independent of temperature so that the entropy vanishes identically.

Finally, it is easy to show that $T_c(p)$, given by equation (50), within the saddle-point approximation scheme, really denotes the temperature at which the two phases CP and SG coexist. This follows immediately by the comparison of the corresponding free energies

$$\frac{F_{SG}}{N}=\frac{F_{CP}}{N} \tag{62}$$

where $F_{SG}/N$ is given by equation (60) for the SG state and for the CP phase one finds [5, 6]

$$\frac{F_{CP}}{N}=-\frac{J^2}{4T}-T\ln 2+\frac{T}{p}\left(\frac{\Gamma}{J}\right)^2. \tag{63}$$

## 5. Concluding remarks

Within the Matsubara time representation, the SG phase of a quantum $p$-spin interaction SG model has been investigated for large but finite $p$ using Parisi's scheme of RSB. Our main aim was not to establish direct contact with the real SG Ising model in a transverse field ($p=2$). Rather, we worked within the more simple picture for large $p$ of the $p$-spin interaction SG model to obtain information about the quantum fluctuation effects in the SG phase, parallel to the corresponding study made in [6] for the paramagnetic state. However, we believe that our present replica procedure, as well as the quantum cavity fields approach [6], which can be developed along the same lines giving similar results, is quite general and can be used for exploring other quantum SG systems taking into account the quantum fluctuations in a natural way.

In the present paper, attention was devoted to the study of the large-$p$ corrections for the known limit $p \to \infty$ [1] which are strictly related to the quantum fluctuations and to their relevance with respect to the SA which is assumed to be valid as $p \to \infty$. An interesting picture emerges: these corrections appear to be much more essential than in the classical counterpart [3]. In particular, in contrast to the tendency found for the classical model for $p < \infty$ where $T_c$ increases as $p$ decreases, in the present problem the quantum fluctuations ($\Gamma \neq 0$) have the effect of depressing the SG transition temperature. In our study, the $\tau$-dependence of the spin autocorrelation function has been obtained in an explicit form for large $p$ in the SG phase and the results for $T$ approaching $T_c$ appear to be quite consistent with those found in [3] for the paramagnetic state. Considering the quantum effects on the global phase diagram of the model under study, we present in figures 1 and 2, as a useful summarizing picture, the schematic $(\Gamma, T)$-phase diagrams when $p = \infty$ and when $p$ is large but finite.

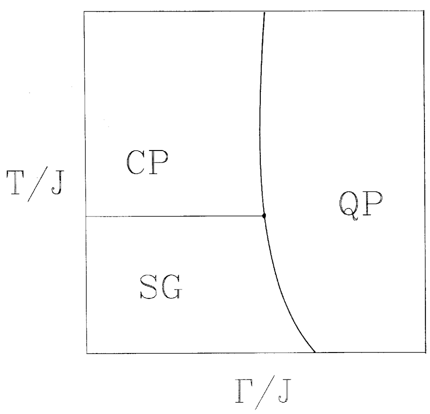


**Figure 1.** A schematic phase diagram for the model (1), (2) when $p = \infty$ (see [4]). There exist three phases, classical paramagnetic (CP), quantum paramagnetic (QP) and spin-glass (SG), separated by coexistence lines. At high temperatures, the CP–QP phase boundary goes to infinity.

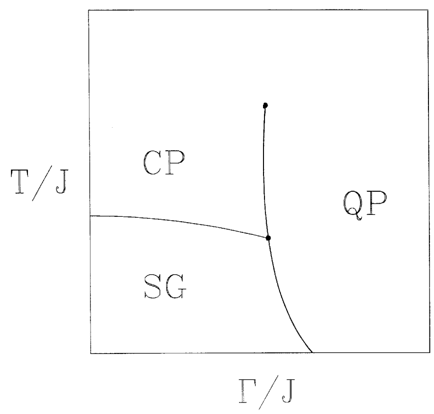


**Figure 2.** A schematic phase diagram for the model (1), (2) when $p$ is large but finite. There again exist the three classical paramagnetic (CP), quantum paramagnetic (QP) and spin-glass (SG) phases separated by coexistence lines, but the CP–QP phase boundary terminates at a critical point (see [5]).

These figures illustrate qualitatively: (i) the difference in the phase diagrams for the two paramagnetic phases between $p = \infty$ and $p \gg 1$, as discussed in the introduction; and (ii) the difference between the CP and SG phase boundaries for $p = \infty$ and $p \gg 1$ as obtained in the present paper. Notice that, in figure 2, the line separating the CP and SG phases sketches part of a parabola according to equations (50) and (51).

Of course, some open problems remain and further deeper studies of the dynamic and static properties of the SG phase are desirable. For instance, an important problem to be further investigated, not only for the quantum model but also for its classical counterpart, is the structure of the SG order parameter function within RSB theory. Indeed, as was seen above, a step function arises from the trivial solution for the sequence of Parisi's parameters $q_l$ $(l = 0, \ldots, k-1)$, but even from preliminary calculations one cannot exclude *a priori* the possibility of the existence of non-trivial solutions with non-zero values of the $q_l$.

Finally, studies of our quantum SG model for finite $p$ and, in particular, as $p \to 2$

(accessible in the classical couterpart [3]) would be particulary interesting but, at present, they appear quite difficult to obtain from the analytical point of view. Of course, numerical information could also be obtained within our scheme but, as already mentioned, this is beyond the purpose of the present paper.

## Acknowledgments

Two of us (KL-W and KW) would like to express their appreciation for the warm hospitality of the Department of Theoretical Physics of Salerno University shown to them during the preparation of this paper. Additional support from the Polish Committee for Scientific Research (KBN), grant No 2 P302 264 03, is gratefully acknowledged.

## Appendix

In this appendix we present a method for calculating $\chi(\tau-\tau')$ given by equation (40) which is particularly effective when $(|\beta - 2|\tau-\tau'|| + \beta_c - \beta) \leqslant 0$.

The relevant $(\tau-\tau')$-dependent part of $\chi(\tau-\tau')$ is (see equation (40) with $\tau-\tau' \to \tau$)

$$\chi^{(1)}(\tau) = \Gamma^2 \frac{B(\tau)}{I_k} \tag{A1}$$

where

$$B(\tau) = \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}} \mathrm{e}^{-x^2/2} \frac{\cosh^{m_k-1}[\beta E_0(x)]}{E_0^2(x)} \cosh[yE_0(x)] \tag{A2}$$

with

$$y = \beta - 2|\tau| \tag{A3}$$

$m_k = \beta_c/\beta$ and $E_0(x)$ is defined by equation (43). The problem is to calculate $B(\tau)$and $I_k$ (see equation (42)) for large $p$. Let us consider the quantities

$$\frac{\partial B}{\partial y} = 2\int_0^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}} \mathrm{e}^{-x^2/2} \frac{\cosh^{m_k-1}[\beta E_0(x)]}{E_0(x)} \sinh[yE_0(x)] \tag{A4}$$

and

$$\frac{\partial^2 B}{\partial y^2} = 2\int_0^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}} \mathrm{e}^{-x^2/2} \cosh^{m_k-1}[\beta E_0(x)] \cosh[yE_0(x)]. \tag{A5}$$

Our idea is to calculate (A5) for large $p$ and then to solve the resulting differential equation for $B$.

For $p \gg 1$ one obtains

$$\frac{\partial^2 B}{\partial y^2} \approx \frac{1}{2^{m_k-1}} \int_0^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2} - (\beta-\beta_c-|y|)J\sqrt{\frac{p}{2}}x\right]. \tag{A6}$$

At this stage, we consider only the case $(\beta-\beta_c-|y|) \geqslant 0$ (for $(\beta-\beta_c-|y|) < 0$, $B$ can be calculated by the saddle-point method), so that, for large $p$,we get

$$\frac{\partial^2 B}{\partial y^2} = \frac{1}{2^{m_k-1}} \frac{1}{\sqrt{\pi p}} \frac{1}{(\beta-\beta_c-|y|)J} + \mathcal{O}(p^{-3/2}). \tag{A7}$$

The solution of the differential equation (A7), with the initial conditions $B(|\tau|)|_{y=0} = B(\beta/2)$ and $\partial B/\partial y|_{y=0} = 0$ (see (A2) and (A4)), is

$$B = \frac{1}{2^{m_k-1}} \frac{1}{\sqrt{\pi p}J} \left[(\beta-\beta_c-|y|)\ln\left(\frac{\beta-\beta_c-|y|}{\beta-\beta_c}\right) + |y| + f(\beta,\Gamma)\right] \tag{A8}$$

with

$$f(\beta, \Gamma) = (2p)^{1/2} J \int_0^\infty \mathrm{d}x \, \frac{\exp[-x^2/2 - (\beta - \beta_\mathrm{c}) J \sqrt{p/2} x]}{\Gamma^2 + pJ^2x^2/2} \tag{A9}$$

for large $p$. Of course, in the limit $p \to \infty$, $f(\beta, \Gamma)$ is finite. In addition, it is easy to show that $B$ has a minimal value at $y = 0$ (or $|\tau| = \beta/2$).

A calculation of $I_k$ by the saddle-point method gives

$$I_k = \frac{1}{2^{m_k - 1}} \exp\left(-\frac{\beta_c^2 J^2 p}{4}\right). \tag{A10}$$

Equations (A8) and (A10) lead immediately to the required result for $\chi^{(1)}(\tau)$ and therefore for $\chi(\tau - \tau')$ through equation (40) in the $(\tau - \tau')$-region of interest.

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